2.1 Intrinsic Carrier Density

The intrinsic carrier density in silicon at room temperature is $1.01 \times 10^{10}\,\mathrm{cm}^{-3}$ ('latest values'). Low field mobilities of electrons and holes are ≈ 1400 and $\approx 480\ \mathrm{cm}^2/(Vs)$, respectively.

1. How many free electrons / holes are present per cubic micrometer?
2. What would be the current flowing through a (ohmically connected) pixel with an area of $200 \times 200\,\mu\mathrm{m}^2$ in a $300\,\mu\mathrm{m}$ thick detector when applying $100\,\mathrm{V}$?
3. How many electrons/holes are that per nanosecond?

2.2 Thickness of Depletion Region

Consider an n-doped wafer of $300\,\mu$m thickness with a bulk resistivity of $2\,\mathrm{k}\Omega\,\mathrm{cm}$. The top surface is p-implanted with 10^{15} atoms per cm^3.

1. What is the bulk wafer doping in atoms per cm^3 and atoms per $\mu\mathrm{m}^3$?
2. What is the build-in voltage?
3. What additional voltage V_{depl} is required to deplete the wafer?
4. What is the field at the pn-junction and at the backside just at depletion?
5. How does the field at the backside increase with extra Over-voltage when the bias voltage is $V_{depl} + V_{over}$?

2.3 Drift in Depletion Region

We want to study in more detail how the charge carriers (electrons, holes) drift though the depletion region by taking into account the *varying* electrical field.

1. For the field we use the expression

$$E(x) = \frac{2V_{\mathrm{depl}}(D-x)}{D^2} + \frac{V_{\mathrm{over}}}{D},$$

   where V_{depl} is the depletion voltage, V_{over} an additional overvoltage and D is the detector thickness. (The junction is at $x = 0$.)
2. Check that $E(x)$ integrates up to $V_{\mathrm{depl}} + V_{\mathrm{over}}$.
3. Plot $E(x)$.
4. The position of a drifting charge obviously depends on time. We want to calculate this $x(t)$. Start with the drift equation $v(t) = \mu \cdot E(x(t))$ and use $x(t)$ to express $v(t)$. Solve the resulting differential equation. You may want to used a mathematical software package for this.

5. Fix the integration constant by the initial condition $x(0) = 0$. If you can, plot the particle position vs. time. You may also include the solution for the naive assumption of a constant field $E_{flat}(x) = (V_{\mathrm{depl}} + V_{\mathrm{over}})/D$.

6. What is the general expression for the time required to reach the backside at $x = D$?

7. What is the drift time for a depletion voltage of 100 V and an overvoltage of 50 V in a $D = 300\,\mu$m thick sensor?

8. If you can, plot the drift time as a function of over-voltage. Plot also the result for the naive assumption $E_{flat}(x)$.

2.4 Linear Depletion

For constant doping densitiy, the depletion region in a diode grows with the square root of the (reverse) bias voltage. In this exercise you should find a (non-constant) doping profile such that the thickness of the depletion region T grows *linearly* with the applied voltage, i.e. such that $T[V] = k \cdot V$. We assume that the junction is at $x = 0$. To the left, we have 'infinite' p-doping. To the right, we assume a n-doping density following a power law

$$n(x) = Ax^{\alpha}.$$

1. Assume that the depletion region extends to $x = T > 0$, i.e. that the donors are depleted and a space charge corresponding to donor density exists. Calculate $E(x)$ from Gauß's law (by integrating over space charge).

2. From $E(x)$, calculate $V(x)$, and in particular $V(T)$.

3. Now find $T(V)$. Check that you find the known result for constant doping.

4. First verify that if you require $T \propto \sqrt{V}$, you find constant doping.

5. What exponent α is required for $T \propto V$? Can this be implemented?