Solution to Exercise: Drift

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Some constants and presets

In[179]:=
```
ClearAll["Global`*"];
```

In[217]:= $\mu e = 1400 \frac{(10^4)^2}{10^9}$ (* mobility in $\frac{\mu m^2}{V\ ns}$ *);

In[181]:=
```
SetOptions[Plot, {Frame → True, Filling → Axis, PlotStyle → {Thickness[0.006]},
    LabelStyle → Directive[Black, 14, FontFamily → "Arial"]}];
```

1. Field Formula

In[182]:=
```
Clear[Vover, Vdepl]; (* Clear old definitions *)
$Assumptions = {Vover > 0 && Vdepl > 0};
```

We know that the field is triangular at Vdepl and has an additional component Vover / D. So we just write it down:

In[184]:= Field[x_, Vover_, Vdepl_] = $\frac{\text{Vover}}{D} + \frac{(D - x)}{D} \frac{2\,\text{Vdepl}}{D}$; (* D in μm, Field in V / um *)

2. Check Integral

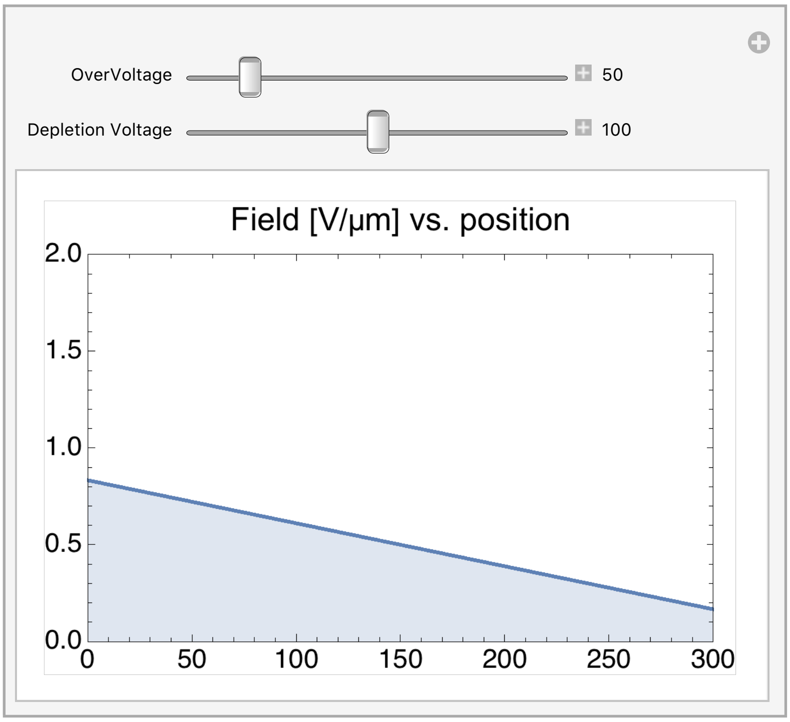
In[185]:=
```
Integrate[Field[x, Vover, Vdepl], {x, 0, D}] == Vdepl + Vover
```

Out[185]=
```
True
```

3. Plot

In[186]:=
```
Manipulate[
 Plot[Field[x, Vover, Vdepl] /. D → 300, {x, 0, 300},
  PlotRange → {0, 2}, PlotLabel → "Field [V/μm] vs. position"]
 , {{Vover, 50, "OverVoltage"}, 0, 400, 10, Appearance → "Labeled"}
 , {{Vdepl, 100, "Depletion Voltage"}, 0, 200, 10, Appearance → "Labeled"}
]
```

Out[186]=



4. Position as a function of time

In[187]:=
```
EQ = x'[t] == μ Field[x[t], Vover, Vdepl] (* v = μ E *)
```

Out[187]= $x'[t] == \mu \left(\frac{\text{Vover}}{D} + \frac{2\,\text{Vdepl}\,(D - x[t])}{D^2}\right)$

In[188]:=
```
DSolve[EQ, x[t], t] // First // FullSimplify
```

Out[188]= $\left\{x[t] \to D + \frac{D\,\text{Vover}}{2\,\text{Vdepl}} + e^{-\frac{2\,t\,\text{Vdepl}\,\mu}{D^2}}\,C[1]\right\}$

In[189]:=
```
xsolgen[t_, Vover_, Vdepl_] = x[t] /. % /. C[1] → C (* The general solution *)
```

Out[189]= $D + C\,e^{-\frac{2\,t\,\text{Vdepl}\,\mu}{D^2}} + \frac{D\,\text{Vover}}{2\,\text{Vdepl}}$

In[190]:=
```
EQ /. {x[t] → xsolgen[t, Vover, Vdepl], x'[t] → D[xsolgen[t, Vover, Vdepl], t]} //
 Simplify (* Check that this solution indeed solves EQ *)
```

Out[190]= True

5. Set Initial Condition x(t=0) == 0 (* start at junction *)

In[191]:=
```
Solve[xsolgen[0, Vover, Vdepl] == 0, C] //
 First (* Find the integration constant from x[0] = 0*)
```

Out[191]= $\left\{C \to -\frac{D\,(2\,\text{Vdepl} + \text{Vover})}{2\,\text{Vdepl}}\right\}$

In[192]:=
```
xsol[t_, Vover_, Vdepl_] = xsolgen[t, Vover, Vdepl] /. % // Simplify
  (* xsol[t] is the solution fulfilling the initial condition *)
```

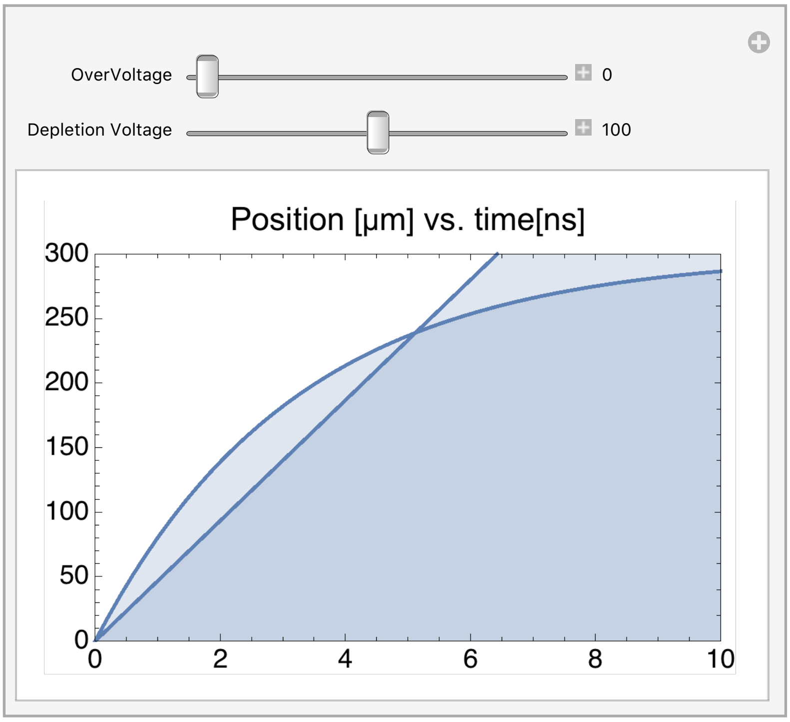
Out[192]=
$$\frac{D\left(2\,\text{Vdepl} + \text{Vover} - e^{-\frac{2\,t\,\text{Vdepl}\,\mu}{D^2}}\,(2\,\text{Vdepl} + \text{Vover})\right)}{2\,\text{Vdepl}}$$

In[193]:=
$$\text{xsolflat}[t\_, \text{Vover}\_, \text{Vdepl}\_] = \mu\,\frac{\text{Vover} + \text{Vdepl}}{D}\,t;$$

```
(* Trivial solution with constant field for comparison *)
```

In[219]:=
```
Manipulate[
 Plot[{xsol[t, Vover, Vdepl], xsolflat[t, Vover, Vdepl]} /. {D → 300, μ → μe}, {t, 0, 10}
  , PlotRange → {0, 300}
  , PlotLabel → "Position [μm] vs. time[ns]"
 ]
 , {{Vover, 0, "OverVoltage"}, 0, 200, 10, Appearance → "Labeled"}
 , {{Vdepl, 100, "Depletion Voltage"}, 0, 200, 10, Appearance → "Labeled"}
]
(* At 0 overvoltage,
the other side at D=300 is never reached because the field is 0 there *)
```

Out[219]=



In[220]:=
$$\text{xsol}[t, \text{Vover}, \text{Vdepl}] == \frac{D\,e^{-\frac{2\,t\,\text{Vdepl}\,\mu}{D^2}}\left(-1 + e^{\frac{2\,t\,\text{Vdepl}\,\mu}{D^2}}\right)(2\,\text{Vdepl} + \text{Vover})}{2\,\text{Vdepl}} \;//\; \text{FullSimplify}$$

Out[220]= True

6. Time to reach Position x in general and backside at x=D when starting at x=0.

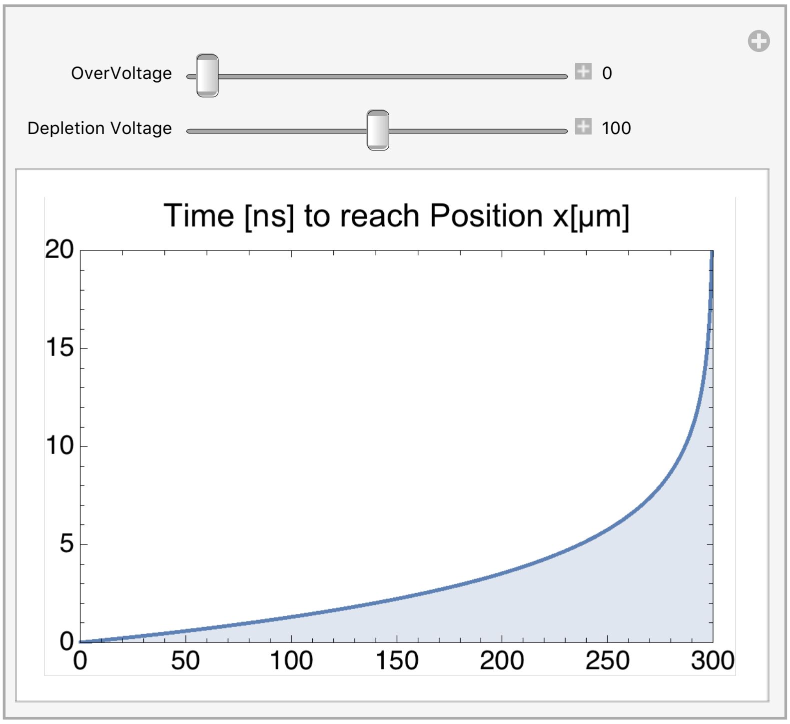
In[221]:=
```
tsol = Solve[x == xsol[t, Vover, Vdepl], t] // First
```

Out[221]=
$$\left\{t \to \text{ConditionalExpression}\left[\frac{D^2\left(2\,i\,\pi\,C[1] + \text{Log}\left[\frac{D\,(2\,\text{Vdepl}+\text{Vover})}{2\,D\,\text{Vdepl}+D\,\text{Vover}-2\,\text{Vdepl}\,x}\right]\right)}{2\,\text{Vdepl}\,\mu},\ C[1] \in \mathbb{Z}\right]\right\}$$

In[222]:=
```
t[x_, Vover_, Vdepl_] = t /. First[Simplify[tsol, C[1] == 0]]
```

Out[222]=
$$\frac{D^2 \,\mathrm{Log}\left[\frac{D\,(2\,\mathrm{Vdepl}+\mathrm{Vover})}{D\,(2\,\mathrm{Vdepl}+\mathrm{Vover})-2\,\mathrm{Vdepl}\,x}\right]}{2\,\mathrm{Vdepl}\,\mu}$$

In[223]:=
```
Manipulate[
 Plot[{t[x, Vover, Vdepl]} /. {D → 300, μ → μe}, {x, 0, 300}, PlotRange → {0, 20}
  , PlotLabel → "Time [ns] to reach Position x[μm]"
 ]
 , {{Vover, 0, "OverVoltage"}, 0, 200, 10, Appearance → "Labeled"}
 , {{Vdepl, 100, "Depletion Voltage"}, 0, 200, 10, Appearance → "Labeled"}
]
```

Out[223]=


In[224]:=
```
tdriftToD = t[D, Vover, Vdepl] // Simplify
```

Out[224]=
$$\frac{D^2 \,\mathrm{Log}\left[1+\frac{2\,\mathrm{Vdepl}}{\mathrm{Vover}}\right]}{2\,\mathrm{Vdepl}\,\mu}$$

In[225]:=
$$\mathrm{tdriftToDflat} = \frac{D\,x}{(\mathrm{Vdepl}+\mathrm{Vover})\,\mu}\ /.\ x \to D;$$

7. Total Drift Time to reach backside at x==D

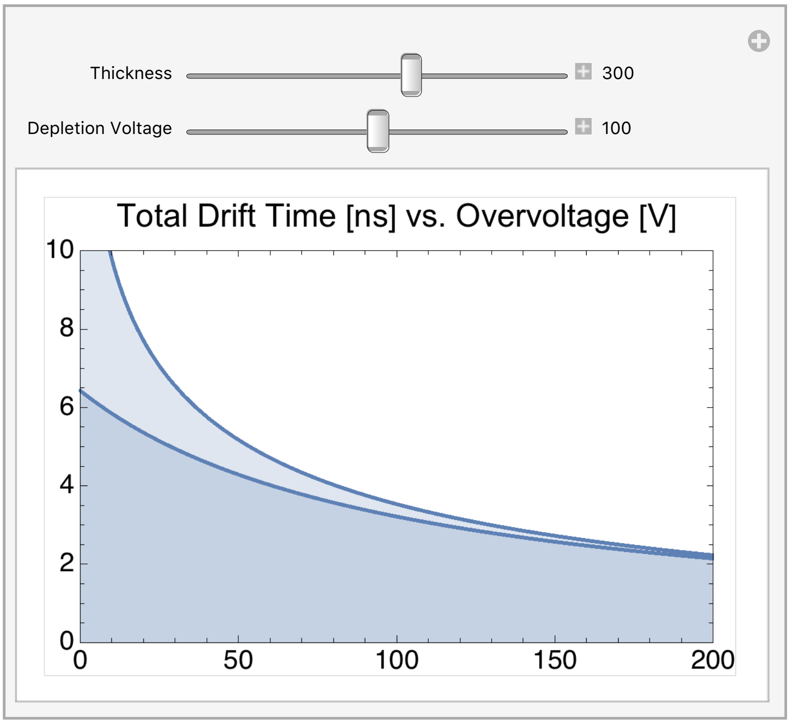
In[226]:=
```
{tdriftToD, tdriftToDflat} /. {Vdepl → 100, Vover → 50, μ → μe, D → 300} // N
```

Out[226]= {5.17319, 4.28571}

8. Plot

In[227]:=
```
Manipulate[
 Plot[{tdriftToD, tdriftToDflat} /. {D → t, Vdepl → b, μ → μe}, {Vover, 0, 200}
  , PlotRange → {0, 10}
  , PlotLabel → "Total Drift Time [ns] vs. Overvoltage [V]"]
 , {{t, 300, "Thickness"}, 0, 500, 50, Appearance → "Labeled"}
 , {{b, 100, "Depletion Voltage"}, 0, 200, 10, Appearance → "Labeled"}
]
```

Out[227]=



Try Calculation with Variable Mobility - No closed solution found..